

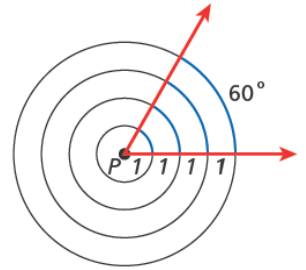
Geometry 12.3 Extension Study Guide: Measuring Angles in radians (pp 816-817)

I can use proportions to convert angle measures from degrees to radians.

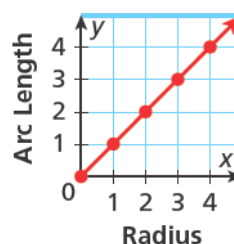
Vocabulary: Radian

One unit of measurement for angles is degrees, which are based on a fraction of a circle. Another unit is called a *radian*, which is based on the relationship of the radius and arc length of a central angle in a circle.

Four concentric circles are shown, with radius 1, 2, 3, and 4. The measure of each arc is 60° .



Radius	Arc Length
1	$2\pi(1)\left(\frac{60^\circ}{360^\circ}\right) = \frac{\pi}{3}$
2	$2\pi(2)\left(\frac{60^\circ}{360^\circ}\right) = \frac{2\pi}{3}$
3	$2\pi(3)\left(\frac{60^\circ}{360^\circ}\right) = \pi$
4	$2\pi(4)\left(\frac{60^\circ}{360^\circ}\right) = \frac{4\pi}{3}$



The relationship between the radius and arc length is linear, with a slope of $2\pi\left(\frac{60^\circ}{360^\circ}\right) = \frac{\pi}{3}$, or about 1.05. The slope represents the ratio of the arc length to the radius. This ratio is the *radian* measure of the angle, so 60° is the same as $\frac{\pi}{3}$ radians.

If a central angle θ in a circle of radius r intercepts an arc of length r , the measure of θ is defined as 1 radian. Since the circumference of a circle of radius r is $2\pi r$, an angle representing one complete rotation measures 2π radians, or 360° .

$$2\pi \text{ radians} = 360^\circ \text{ and } \pi \text{ radians} = 180^\circ$$

$$1^\circ = \frac{\pi \text{ radians}}{180^\circ} \text{ and } 1 \text{ radian} = \frac{180^\circ}{\pi \text{ radians}}$$

Use these facts to convert between radians and degrees.

Converting Angle Measures

DEGREES TO RADIANS	RADIANS TO DEGREES
Multiply the number of degrees by	Multiply the number of radians by
$\left(\frac{\pi \text{ radians}}{180^\circ}\right)$	$\left(\frac{180^\circ}{\pi \text{ radians}}\right)$

Remember!

Arc length is the distance along an arc measured in linear units. In a circle of radius r , the length of an arc with a central angle measure m is $L = 2\pi r \left(\frac{m^\circ}{360^\circ}\right)$

1 Converting Degrees to Radians

Convert each measure from degrees to radians.

Multiply by $\left(\frac{\pi \text{ radians}}{180^\circ}\right)$.

A 30°

$$\cancel{30}^\circ \left(\frac{\pi \text{ radians}}{\cancel{180}^\circ_6}\right) = \frac{\pi}{6} \text{ radians}$$

B 75°

$$\cancel{75}^\circ \left(\frac{\pi \text{ radians}}{\cancel{180}^\circ_{12}}\right) = \frac{5\pi}{12} \text{ radians}$$

Example 1. Convert each measure from degrees to radians.

A. 85°

B. 90°

Guided Practice. Convert each measure from degrees to radians.

1. -36°

2. 270°

(p 817) 1, 5, 7,

Helpful Hint

Because the radian measure of an angle is related to arc length, the most commonly used angle measures are usually written as fractional multiples of π .

2 Converting Radians to Degrees

Convert each measure from radians to degrees.

A $\frac{\pi}{4}$ radians

$$\frac{\pi}{4} \text{ radians} \left(\frac{45}{180^\circ} \right) = 45^\circ$$

B $\frac{2\pi}{9}$ radians *Multiply by $\left(\frac{180^\circ}{\pi \text{ radians}}\right)$.*

$$\frac{2\pi}{9} \text{ radians} \left(\frac{20}{180^\circ} \right) = 40^\circ$$

Example 2. Convert each measure from radians to degrees.

A. $\frac{2\pi}{3}$ radians

B. $\frac{\pi}{6}$ radians

Guided Practice. Convert each measure from radians to degrees.

3. $\frac{5\pi}{6}$ radians

4. $-\frac{3\pi}{4}$ radians

12.3 Extension Assignment

- (p 817) 1, 5, 7, 13, 17-20
- Ready to Go On Section 8b

